

Package ‘gasmodel’

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Type Package

Title Generalized Autoregressive Score Models

Version 0.3.0

Description Estimation, forecasting, and simulation of generalized autoregressive score (GAS) models of Creal, Koopman, and Lucas (2013) <[doi:10.1002/jae.1279](https://doi.org/10.1002/jae.1279)> and Harvey (2013) <[doi:10.1017/cbo9781139540933](https://doi.org/10.1017/cbo9781139540933)>. Model specification allows for various conditional distributions, different parametrizations, exogenous variables, joint and separate modeling of exogenous variables and dynamics, higher score and autoregressive orders, custom and unconditional initial values of time-varying parameters, fixed and bounded values of coefficients, and missing values. Model estimation is performed by the maximum likelihood method and the Hessian matrix.

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BugReports <https://github.com/vladimirholy/gasmodel/issues>

NeedsCompilation no

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bookshop_sales	<i>Antiquarian Bookshop Sales</i>
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Description

Individual orders of a Czech antiquarian bookshop from June 8, 2018 to December 20, 2018. This dataset is analyzed in Tomanová and Holý (2021). Details on the bookshop can be also found in Tomanová and Černý (2022).

Usage

bookshop_sales

Format

A data frame with columns:

order ID of the order.

time Time of the order.

quantity Number of purchased books. Zero value means the order was canceled.

Source

Petra Tomanová (<petra.tomanova@vse.cz>).

References

Tomanová, P. and Černý, M. (2022). Efficiency of Antiquarian Bookshops in Informationally Complete Markets. *Central European Journal of Operations Research*, **30**(2), 573–593. doi: [10.1007/s10100021007803](https://doi.org/10.1007/s10100021007803).

Tomanová, P. and Holý, V. (2021). Clustering of Arrivals in Queueing Systems: Autoregressive Conditional Duration Approach. *Central European Journal of Operations Research*, **29**(3), 859–874. doi: [10.1007/s10100021007447](https://doi.org/10.1007/s10100021007447).

distr

Get the Table of Supported Distributions

Description

A function listing distributions and their parametrizations supported by the `gas()` function. Output can be filtered using several arguments.

Usage

```
distr(filter_distr = NULL, filter_param = NULL, filter_type = NULL,
      filter_dim = NULL, filter_orthog = NULL, filter_default = NULL)
```

Arguments

<code>filter_distr</code>	An optional vector of distributions by which the output is filtered.
<code>filter_param</code>	An optional vector of parametrizations by which the output is filtered.
<code>filter_type</code>	An optional vector of data types by which the output is filtered.
<code>filter_dim</code>	An optional vector of dimensions by which the output is filtered.
<code>filter_orthog</code>	An optional logical value indicating whether the parametrization is orthogonal by which the output is filtered.
<code>filter_default</code>	An optional logical value indicating whether the parameterization is the default for the distribution by which the output is filtered.

Value

A data.frame with columns:

<code>distr_title</code>	The title of the distribution.
<code>param_title</code>	The title of the parametrization.
<code>distr</code>	The distribution.
<code>param</code>	The parametrization.
<code>type</code>	The data type.
<code>dim</code>	The dimension.
<code>orthog</code>	The indication of whether the parametrization is orthogonal.
<code>default</code>	The indication of whether the parameterization is the default for the distribution.

See Also

[distr_density\(\)](#), [distr_mean\(\)](#), [distr_var\(\)](#), [distr_score\(\)](#), [distr_fisher\(\)](#), [distr_random\(\)](#), [gas\(\)](#)

Examples

```
# List all available distributions
distr()

# List only distributions for count data
distr(filter_type = "count")

# Show default parametrization for the exponential distribution
distr(filter_dist = "exp", filter_default = TRUE)
```

distr_density	<i>Compute Density</i>
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Description

A function computing density or its logarithm of a given distribution.

Usage

```
distr_density(y, f, distr, param = NULL, par_link = NULL, trans = NULL)
```

Arguments

y	Observations. For an univariate distribution, a numeric vector. For a multivariate distribution, a numeric matrix with observations in rows or a numeric vector of a single observation.
f	Parameters. For the same parameters for all observations, a numeric vector. For individual parameters for each observation, a numeric matrix with rows corresponding to observations.
distr	A distribution.
param	A parametrization of the distribution.
par_link	An optional logical vector indicating whether the logarithmic/logistic link should be applied to restricted parameters in order to obtain unrestricted values. Defaults to keeping the original link for all parameters.
trans	An optional transformation of the density. The supported transformation is the logarithm of the density (<code>trans = "log"</code>).

Value

The (transformed) density.

See Also[distr\(\)](#)**Examples**

```
# Density of the negative binomial distribution
distr_density(y = c(1, 8, 5, 0, 0), f = c(0.5, 1.2), distr = "negbin")

# Density of the multivariate normal distribution
distr_density(y = rbind(c(0.5, 0.6), c(-2.3, -1.8), c(-0.2, 0.2)),
              f = c(0, 0, 1, 1, 0.5), distr = "mvnorm")
```

distr_fisher	<i>Compute Fisher Information</i>
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Description

A function computing Fisher information, its inverse, or its inverse square root for a given distribution.

Usage

```
distr_fisher(f, distr, param = NULL, par_link = NULL, trans = NULL)
```

Arguments

f	Parameters. For the same parameters for all observations, a numeric vector. For individual parameters for each observation, a numeric matrix with rows corresponding to observations.
distr	A distribution.
param	A parametrization of the distribution.
par_link	An optional logical vector indicating whether the logarithmic/logistic link should be applied to restricted parameters in order to obtain unrestricted values. Defaults to keeping the original link for all parameters.
trans	An optional transformation of the Fisher information. The supported transformations are the inverse of the Fisher information (<code>trans = "inv"</code>) and the inverse square root of the Fisher information (<code>trans = "inv_sqrt"</code>).

Value

The (transformed) Fisher information.

See Also[distr\(\)](#)

Examples

```
# Fisher information for the negative binomial distribution
distr_fisher(f = c(0.5, 1.2), distr = "negbin")

# Fisher information for the multivariate normal distribution
distr_fisher(f = c(0, 0, 1, 1, 0.5), distr = "mvnorm")
```

distr_mean

Compute Mean

Description

A function computing mean for a given distribution.

Usage

```
distr_mean(f, distr, param = NULL, par_link = NULL)
```

Arguments

f	Parameters. For the same parameters for all observations, a numeric vector. For individual parameters for each observation, a numeric matrix with rows corresponding to observations.
distr	A distribution.
param	A parametrization of the distribution.
par_link	An optional logical vector indicating whether the logarithmic/logistic link should be applied to restricted parameters in order to obtain unrestricted values. Defaults to keeping the original link for all parameters.

Value

The mean.

See Also

[distr\(\)](#)

Examples

```
# Mean for the negative binomial distribution
distr_mean(f = c(0.5, 1.2), distr = "negbin")

# Mean for the multivariate normal distribution
distr_mean(f = c(0, 0, 1, 1, 0.5), distr = "mvnorm")
```

distr_random	<i>Generate Random Observations</i>
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Description

A function generating random observations from a given distribution.

Usage

```
distr_random(t, f, distr, param = NULL, par_link = NULL)
```

Arguments

t	A number of generated observations.
f	A numeric vector of parameters. The same parameters are used for each generated observation.
distr	A distribution.
param	A parametrization of the distribution.
par_link	An optional logical vector indicating whether the logarithmic/logistic link should be applied to restricted parameters in order to obtain unrestricted values. Defaults to keeping the original link for all parameters.

Value

The generated observations.

See Also

[distr\(\)](#)

Examples

```
# Random observations from the negative binomial distribution
distr_random(t = 10, f = c(0.5, 1.2), distr = "negbin")

# Random observations from the multivariate normal distribution
distr_random(t = 10, f = c(0, 0, 1, 1, 0.5), distr = "mvnorm")
```

distr_score *Compute Score*

Description

A function computing score or scaled score for a given distribution.

Usage

```
distr_score(y, f, distr, param = NULL, par_link = NULL, scaling = NULL)
```

Arguments

y	Observations. For an univariate distribution, a numeric vector. For a multivariate distribution, a numeric matrix with observations in rows or a numeric vector of a single observation.
f	Parameters. For the same parameters for all observations, a numeric vector. For individual parameters for each observation, a numeric matrix with rows corresponding to observations.
distr	A distribution.
param	A parametrization of the distribution.
par_link	An optional logical vector indicating whether the logarithmic/logistic link should be applied to restricted parameters in order to obtain unrestricted values. Defaults to keeping the original link for all parameters.
scaling	An optional scaling function for the score. The supported scaling functions are the unit scaling (<code>scaling = "unit"</code>), the inverse of the Fisher information matrix scaling (<code>scaling = "fisher_inv"</code>), and the inverse square root of the Fisher information matrix scaling (<code>scaling = "fisher_inv_sqrt"</code>).

Value

The (scaled) score.

See Also

[distr\(\)](#)

Examples

```
# Score for the negative binomial distribution
distr_score(y = c(1, 8, 5, 0, 0), f = c(0.5, 1.2), distr = "negbin")

# Score for the multivariate normal distribution
distr_score(y = rbind(c(0.5, 0.6), c(-2.3, -1.8), c(-0.2, 0.2)),
             f = c(0, 0, 1, 1, 0.5), distr = "mvnorm")
```

distr_var	<i>Compute Variance</i>
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Description

A function computing variance for a given distribution.

Usage

```
distr_var(f, distr, param = NULL, par_link = NULL)
```

Arguments

f	Parameters. For the same parameters for all observations, a numeric vector. For individual parameters for each observation, a numeric matrix with rows corresponding to observations.
distr	A distribution.
param	A parametrization of the distribution.
par_link	An optional logical vector indicating whether the logarithmic/logistic link should be applied to restricted parameters in order to obtain unrestricted values. Defaults to keeping the original link for all parameters.

Value

The variance.

See Also

[distr\(\)](#)

Examples

```
# Variance for the negative binomial distribution
distr_var(f = c(0.5, 1.2), distr = "negbin")

# Variance for the multivariate normal distribution
distr_var(f = c(0, 0, 1, 1, 0.5), distr = "mvnorm")
```

gas

*Estimate GAS Model***Description**

A versatile function for estimation of generalized autoregressive score (GAS) models of Creal et al. (2013) and Harvey (2013). Model specification allows for various conditional distributions, different parametrizations, exogenous variables, higher score and autoregressive orders, custom and unconditional initial values of time-varying parameters, fixed and bounded values of coefficients, and NA values. Model estimation is performed by the maximum likelihood method and the Hessian matrix. The function can be supplied with any optimization and Hessian functions.

Usage

```
gas(y, x = NULL, distr, param = NULL, scaling = "unit", spec = "joint",
    p = 1L, q = 1L, par_static = NULL, par_link = NULL,
    par_init = NULL, lik_skip = 0L, coef_fix_value = NULL,
    coef_fix_other = NULL, coef_fix_special = NULL,
    coef_bound_lower = NULL, coef_bound_upper = NULL, coef_start = NULL,
    optim_function = wrapper_optim_nloptr, optim_arguments = list(opts =
    list(algorithm = "NLOPT_LN_NELDERMEAD", xtol_rel = 0, maxeval = 1e+06)),
    hessian_function = wrapper_hessian_stats, hessian_arguments = list(),
    print_progress = FALSE)
```

Arguments

y	A time series. For univariate time series, a numeric vector or a matrix with a single column. For multivariate times series, a numeric matrix with observations in rows.
x	Optional exogenous variables. For a single variable common for all time-varying parameters, a numeric vector. For multiple variables common for all time-varying parameters, a numeric matrix with observations in rows. For individual variables for each time-varying parameter, a list of numeric vectors or matrices in the above form. The number of observation must be equal to the number of observations of y.
distr	A conditional distribution. See distr() for available distributions.
param	A parametrization of the conditional distribution. If NULL, default parametrization is used. See distr() for available parametrizations.
scaling	A scaling function for the score. The supported scaling functions are the unit scaling (<code>scaling = "unit"</code>), the inverse of the Fisher information matrix scaling (<code>scaling = "fisher_inv"</code>), and the inverse square root of the Fisher information matrix scaling (<code>scaling = "fisher_inv_sqrt"</code>). The latter two scalings use the Fisher information for the time-varying parameters only. For the full Fisher information matrix for both time-varying and static parameters, there are the <code>"full_fisher_inv"</code> and <code>"full_fisher_inv_sqrt"</code> scalings. For the individual Fisher information for each parameter, there are the <code>"diag_fisher_inv"</code>

and "diag_fisher_inv_sqrt" scalings. Note that when the parametrization is orthogonal (see `distr()`), there are no differences between these scaling variants.

spec	A specification of the dynamic equation with regard to exogenous variables. The supported specifications are exogenous variables and dynamics within the same equation (spec = "joint") and separate equations for exogenous variables and dynamics in the fashion of regression models with dynamic errors (spec = "reg_err"). In a stationary model without exogenous variables, the two specifications are equivalent, although with differently parametrized intercept.
p	A score order. For order common for all parameters, a numeric vector of length 1. For individual order for each parameter, a numeric vector of length equal to the number of parameters. Defaults to 1L.
q	An autoregressive order. For order common for all parameters, a numeric vector of length 1. For individual order for each parameter, a numeric vector of length equal to the number of parameters. Defaults to 1L.
par_static	An optional logical vector indicating static parameters. Overrides x, p, and q.
par_link	An optional logical vector indicating whether the logarithmic/logistic link should be applied to restricted parameters in order to obtain unrestricted values. Defaults to applying the logarithmic/logistic link for time-varying parameters and keeping the original link for constant parameters.
par_init	An optional numeric vector of initial values of time-varying parameters. For NA values or when NULL, set initial values to unconditional values of time-varying parameters. For example, in the case of GAS(1,1) model with spec = "joint", to $\omega / (1 - \phi_1)$. Not to be confused with starting values for the optimization <code>coef_start</code> .
lik_skip	A numeric value specifying the number of skipped observations at the beginning of the time series or after NA values in the likelihood computation. Defaults to 0L, i.e. the full likelihood. If NULL, it is selected as $\max(p, q)$, i.e. the conditional likelihood.
coef_fix_value	An optional numeric vector of values to which coefficients are to be fixed. NA values represent coefficients to be estimated.
coef_fix_other	An optional square numeric matrix of multiples of the estimated coefficients, which are to be added to the fixed coefficients. This allows the fixed coefficients to be linear combinations of the estimated coefficients. A coefficient given by row is fixed on coefficient given by column. By this logic, all rows corresponding to the estimated coefficients should contain only NA values. Furthermore, all columns corresponding to the fixed coefficients should also contain only NA values.
coef_fix_special	An optional character vector of predefined structures of <code>coef_fix_value</code> and <code>coef_fix_other</code> . Useful mainly for multidimensional models. Value "panel_structure" forces all regression, autoregression, and score coefficients to be the same for all time-varying parameters within their group. Value "zero_sum_intercept" forces all constant parameters to sum up to zero within their group. Value "random_walk" forces all autoregressive coefficients to be equal to one (should

be used with caution due to nonstationarity; `par_init` must be specified). Multiple predefined structures can be used together. Also can be used in combination with custom `coef_fix_value` and `coef_fix_other`.

<code>coef_bound_lower</code>	An optional numeric vector of lower bounds on coefficients.
<code>coef_bound_upper</code>	An optional numeric vector of upper bounds on coefficients.
<code>coef_start</code>	An optional numeric vector of starting values for the optimization. If not supplied, starting values are selected from a small grid of values.
<code>optim_function</code>	An optimization function. For suitable wrappers of common R optimization functions, see wrappers_optim . Can be set to NULL if the optimal solution should not be computed, which can be useful if the goal is only to evaluate the fit for the coefficients specified in argument <code>coef_start</code> .
<code>optim_arguments</code>	An optional list of arguments to be passed to the optimization function.
<code>hessian_function</code>	A Hessian function. For suitable wrappers of common R Hessian functions, see wrappers_hessian . Can be set to NULL if the Hessian matrix should not be computed, which can speed up computations when asymptotic inference is not desired.
<code>hessian_arguments</code>	An optional list of arguments to be passed to the Hessian function.
<code>print_progress</code>	A logical value indicating whether to progressively print a detailed report on computation.

Details

The generalized autoregressive score (GAS) models of Creal et al. (2013) and Harvey (2013), also known as dynamic conditional score (DCS) models or score-driven (SD) models, have established themselves as a useful modern framework for time series modeling.

The GAS models are observation-driven models allowing for any underlying probability distribution $p(y_t|f_t)$ with any time-varying parameters f_t for time series y_t . They capture the dynamics of time-varying parameters using the autoregressive term and the lagged score, i.e. the gradient of the log-likelihood function. Exogenous variables can also be included. Specifically, time-varying parameters f_t follow the recursion

$$f_t = \omega + \sum_{i=1}^M \beta_i x_{ti} + \sum_{j=1}^P \alpha_j S(f_{t-j}) \nabla(y_{t-j}, f_{t-j}) + \sum_{k=1}^Q \varphi_k f_{t-k},$$

where ω is a vector of constants, β_i are regression parameters, α_j are score parameters, φ_k are autoregressive parameters, x_{ti} are exogenous variables, $S(f_t)$ is a scaling function for the score, and $\nabla(y_t, f_t)$ is the score given by

$$\nabla(y_t, f_t) = \frac{\partial \ln p(y_t|f_t)}{\partial f_t}.$$

Alternatively, a different model can be obtained by defining the recursion in the fashion of regression models with dynamic errors as

$$f_t = \omega + \sum_{i=1}^M \beta_i x_{ti} + e_t, \quad e_t = \sum_{j=1}^P \alpha_j S(f_{t-j}) \nabla(y_{t-j}, f_{t-j}) + \sum_{k=1}^Q \varphi_k e_{t-k}.$$

The GAS models can be straightforwardly estimated by the maximum likelihood method. For the asymptotic theory regarding the GAS models and maximum likelihood estimation, see Blasques et al. (2014), Blasques et al. (2018), and Blasques et al. (2022).

The use of the score for updating time-varying parameters is optimal in an information theoretic sense. For an investigation of the optimality properties of GAS models, see Blasques et al. (2015) and Blasques et al. (2021).

Generally, the GAS models perform quite well when compared to alternatives, including parameter-driven models. For a comparison of the GAS models to alternative models, see Koopman et al. (2016) and Blazsek and Licht (2020).

The GAS class includes many well-known econometric models, such as the generalized autoregressive conditional heteroskedasticity (GARCH) model of Bollerslev (1986), the autoregressive conditional duration (ACD) model of Engle and Russel (1998), and the Poisson count model of Davis et al. (2003). More recently, a variety of novel score-driven models has been proposed, such as the Beta-t-(E)GARCH model of Harvey and Chakravarty (2008), the discrete price changes model of Koopman et al. (2018), the circular model of Harvey (2019), the bivariate Poisson model of Koopman and Lit (2019), and the ranking model of Holý and Zouhar (2022). For an overview of various GAS models, see Harvey (2022).

The extensive GAS literature is listed on www.gasmodel.com.

Value

A list of S3 class `gas` with components:

<code>data\$y</code>	The time series.
<code>data\$x</code>	The exogenous variables.
<code>model\$distr</code>	The conditional distribution.
<code>model\$param</code>	The parametrization of the conditional distribution.
<code>model\$scaling</code>	The scaling function.
<code>model\$spec</code>	The specification of the dynamic equation.
<code>model\$t</code>	The length of the time series.
<code>model\$n</code>	The dimension of the model.
<code>model\$m</code>	The number of exogenous variables.
<code>model\$p</code>	The score order.
<code>model\$q</code>	The autoregressive order.
<code>model\$par_static</code>	The static parameters.
<code>model\$par_link</code>	The parameters with the logarithmic/logistic links.

`model$par_init` The initial values of the time-varying parameters.
`model$lik_skip` The number of skipped observations at the beginning of the time series or after NA values in the likelihood computation.
`model$coef_fix_value`
 The values to which coefficients are fixed.
`model$coef_fix_other`
 The multiples of the estimated coefficients, which are added to the fixed coefficients.
`model$coef_fix_special`
 The predefined structures of `coef_fix_value` and `coef_fix_other`.
`model$coef_bound_lower`
 The lower bounds on coefficients.
`model$coef_bound_upper`
 The upper bounds on coefficients.
`model$num_obs` The actual number of observations used in the likelihood.
`model$num_coef` The actual number of estimated coefficients.
`control$optim_function`
 The optimization function.
`control$optim_arguments`
 The arguments which are passed to the optimization function.
`control$hessian_function`
 The Hessian function.
`control$hessian_arguments`
 The arguments which are passed to the Hessian function.
`solution$status_start`
 The status of the starting values computation.
`solution$theta_start`
 The computed starting values.
`solution$status_optim`
 The status of the optimization computation.
`solution$theta_optim`
 The computed optimal values.
`solution$status_hessian`
 The status of the Hessian computation.
`solution$theta_hessian`
 The computed Hessian.
`fit$coef_est` The estimated coefficients.
`fit$coef_vcov` The estimated variance-covariance matrix.
`fit$coef_sd` The estimated standard deviations.
`fit$coef_zstat` The statistics of the Z-test.
`fit$coef_pval` The p-values of the Z-test.
`fit$par_unc` The unconditional values of time-varying parameters.
`fit$par_tv` The individual values of time-varying parameter.

<code>fit\$score_tv</code>	The individual scores of time-varying parameters.
<code>fit\$mean_tv</code>	The expected values given by the model.
<code>fit\$var_tv</code>	The variances given by the model.
<code>fit\$resid_tv</code>	The residuals of the model.
<code>fit\$loglik_tv</code>	The log-likelihoods for the individual observations.
<code>fit\$loglik_sum</code>	The overall log-likelihood.
<code>fit\$aic</code>	The Akaike information criterion.
<code>fit\$bic</code>	The Bayesian information criterion.

Note

Supported generic functions for S3 class `gas` include `coef()`, `vcov()`, `residuals()`, `logLik()`, `AIC()`, `BIC()`, and `confint()`.

References

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See Also

[distr\(\)](#), [gas_bootstrap\(\)](#), [gas_filter\(\)](#), [gas_forecast\(\)](#), [gas_simulate\(\)](#), [wrappers_optim](#), [wrappers_hessian](#)

Examples

```
# Load Level of Lake Huron dataset
data(LakeHuron)
y <- LakeHuron - 570
x <- 1:length(y)

# Estimate GAS model based on the normal distribution with dynamic mean
est_gas <- gas(y = y, x = x, distr = "norm", spec = "reg_err",
  par_static = c(FALSE, TRUE), coef_start = c(9.99, -0.02, 0.46, 0.67, 0.46))
est_gas

# Obtain the estimated coefficients
coef(est_gas)

# Obtain the estimated variance-covariance matrix
vcov(est_gas)

# Obtain the log-likelihood, AIC, and BIC
logLik(est_gas)
AIC(est_gas)
BIC(est_gas)

# Obtain the confidence intervals of coefficients
confint(est_gas)
```

gasmodel

gasmodel: Generalized Autoregressive Score Models

Description

This package offers tools for estimation, forecasting, and simulation of generalized autoregressive score (GAS) models of Creal et al. (2013) and Harvey (2013), also known as dynamic conditional score (DCS) models or score-driven (SD) models.

The key function is `gas()` which estimates GAS models. Additional functions include `gas_simulate()` which simulates GAS models, `gas_forecast()` which forecasts GAS models, `gas_filter()` which obtains filtered time-varying parameters of GAS models, and `gas_bootstrap()` which bootstraps coefficients of GAS models.

The list of supported distributions can be obtained by `distr()`. The functions working with distributions are `distr_density()` which computes the density, `distr_mean()` which computes the mean, `distr_var()` which computes the variance, `distr_score()` which computes the score, `distr_fisher()` which computes the Fisher information, and `distr_random()` which generates random observations.

The included datasets are `bookshop_sales` which contains times of antiquarian bookshop sales, `ice_hockey_championships` which contains the results of the Ice Hockey World Championships, and `sp500_daily` which contains daily S&P 500 prices.

References

Creal, D., Koopman, S. J., and Lucas, A. (2013). Generalized Autoregressive Score Models with Applications. *Journal of Applied Econometrics*, **28**(5), 777–795. doi: [10.1002/jae.1279](https://doi.org/10.1002/jae.1279).

Harvey, A. C. (2013). *Dynamic Models for Volatility and Heavy Tails: With Applications to Financial and Economic Time Series*. Cambridge University Press. doi: [10.1017/cbo9781139540933](https://doi.org/10.1017/cbo9781139540933).

gas_bootstrap

Bootstrap GAS Model

Description

A function for bootstrapping coefficients of generalized autoregressive score (GAS) models of Creal et al. (2013) and Harvey (2013). Method "parametric" repeatedly simulates time series using the parametric model and re-estimates coefficients. Instead of supplying arguments about the model, the function can be applied to the `gas` object obtained by the `gas()` function.

Usage

```
gas_bootstrap(gas_object = NULL, method = "parametric", rep_boot = 1000L,
  quant = c(0.025, 0.975), y = NULL, x = NULL, distr = NULL,
  param = NULL, scaling = "unit", spec = "joint", p = 1L, q = 1L,
  par_static = NULL, par_link = NULL, par_init = NULL, lik_skip = 0L,
  coef_fix_value = NULL, coef_fix_other = NULL, coef_fix_special = NULL,
  coef_bound_lower = NULL, coef_bound_upper = NULL, coef_est = NULL,
  optim_function = wrapper_optim_nloptr, optim_arguments = list(opts =
  list(algorithm = "NLOPT_LN_NELDERMEAD", xtol_rel = 0, maxeval = 1e+06)))
```

Arguments

gas_object	An optional GAS estimate, i.e. a list of S3 class gas returned by function <code>gas()</code> .
method	A method used for bootstrapping. Currently, the only supported method is "parametric".
rep_boot	A number of bootstrapping repetitions.
quant	A numeric vector of probabilities determining quantiles.
y	A time series. For univariate time series, a numeric vector or a matrix with a single column. For multivariate times series, a numeric matrix with observations in rows.
x	Optional exogenous variables. For a single variable common for all time-varying parameters, a numeric vector. For multiple variables common for all time-varying parameters, a numeric matrix with observations in rows. For individual variables for each time-varying parameter, a list of numeric vectors or matrices in the above form. The number of observation must be equal to the number of observations of y.
distr	A conditional distribution. See <code>distr()</code> for available distributions.
param	A parametrization of the conditional distribution. If NULL, default parametrization is used. See <code>distr()</code> for available parametrizations.
scaling	A scaling function for the score. The supported scaling functions are the unit scaling (<code>scaling = "unit"</code>), the inverse of the Fisher information matrix scaling (<code>scaling = "fisher_inv"</code>), and the inverse square root of the Fisher information matrix scaling (<code>scaling = "fisher_inv_sqrt"</code>). The latter two scalings use the Fisher information for the time-varying parameters only. For the full Fisher information matrix for both time-varying and static parameters, there are the "full_fisher_inv" and "full_fisher_inv_sqrt" scalings. For the individual Fisher information for each parameter, there are the "diag_fisher_inv" and "diag_fisher_inv_sqrt" scalings. Note that when the parametrization is orthogonal (see <code>distr()</code>), there are no differences between these scaling variants.
spec	A specification of the dynamic equation with regard to exogeneous variables. The supported specifications are exogenous variables and dynamics within the same equation (<code>spec = "joint"</code>) and separate equations for exogenous variables and dynamics in the fashion of regression models with dynamic errors (<code>spec = "reg_err"</code>). In a stationary model without exogenous variables, the two specifications are equivalent, although with differently parametrized intercept.

p	A score order. For order common for all parameters, a numeric vector of length 1. For individual order for each parameter, a numeric vector of length equal to the number of parameters. Defaults to 1L.
q	An autoregressive order. For order common for all parameters, a numeric vector of length 1. For individual order for each parameter, a numeric vector of length equal to the number of parameters. Defaults to 1L.
par_static	An optional logical vector indicating static parameters. Overrides x, p, and q.
par_link	An optional logical vector indicating whether the logarithmic/logistic link should be applied to restricted parameters in order to obtain unrestricted values. Defaults to applying the logarithmic/logistic link for time-varying parameters and keeping the original link for constant parameters.
par_init	An optional numeric vector of initial values of time-varying parameters. For NA values or when NULL, set initial values to unconditional values of time-varying parameters. For example, in the case of GAS(1,1) model with spec = "joint", to $\omega / (1 - \phi_1)$. Not to be confused with starting values for the optimization coef_start.
lik_skip	A numeric value specifying the number of skipped observations at the beginning of the time series or after NA values in the likelihood computation. Defaults to 0L, i.e. the full likelihood. If NULL, it is selected as $\max(p, q)$, i.e. the conditional likelihood.
coef_fix_value	An optional numeric vector of values to which coefficients are to be fixed. NA values represent coefficients to be estimated.
coef_fix_other	An optional square numeric matrix of multiples of the estimated coefficients, which are to be added to the fixed coefficients. This allows the fixed coefficients to be linear combinations of the estimated coefficients. A coefficient given by row is fixed on coefficient given by column. By this logic, all rows corresponding to the estimated coefficients should contain only NA values. Furthermore, all columns corresponding to the fixed coefficients should also contain only NA values.
coef_fix_special	An optional character vector of predefined structures of coef_fix_value and coef_fix_other. Useful mainly for multidimensional models. Value "panel_structure" forces all regression, autoregression, and score coefficients to be the same for all time-varying parameters within their group. Value "zero_sum_intercept" forces all constant parameters to sum up to zero within their group. Value "random_walk" forces all autoregressive coefficients to be equal to one (should be used with caution due to nonstationarity; par_init must be specified). Multiple predefined structures can be used together. Also can be used in combination with custom coef_fix_value and coef_fix_other.
coef_bound_lower	An optional numeric vector of lower bounds on coefficients.
coef_bound_upper	An optional numeric vector of upper bounds on coefficients.
coef_est	A numeric vector of estimated coefficients.
optim_function	An optimization function. For suitable wrappers of common R optimization functions, see wrappers_optim . Can be set to NULL if the optimal solution

should not be computed, which can be useful if the goal is only to evaluate the fit for the coefficients specified in argument `coef_start`.

`optim_arguments`

An optional list of arguments to be passed to the optimization function.

Value

A list with components:

<code>data\$y</code>	The time series.
<code>data\$x</code>	The exogenous variables.
<code>model\$distr</code>	The conditional distribution.
<code>model\$param</code>	The parametrization of the conditional distribution.
<code>model\$scaling</code>	The scaling function.
<code>model\$spec</code>	The specification of the dynamic equation.
<code>model\$t</code>	The length of the time series.
<code>model\$n</code>	The dimension of the model.
<code>model\$m</code>	The number of exogenous variables.
<code>model\$p</code>	The score order.
<code>model\$q</code>	The autoregressive order.
<code>model\$par_static</code>	The static parameters.
<code>model\$par_link</code>	The parameters with the logarithmic/logistic links.
<code>model\$par_init</code>	The initial values of the time-varying parameters.
<code>model\$lik_skip</code>	The number of skipped observations at the beginning of the time series or after NA values in the likelihood computation.
<code>model\$coef_fix_value</code>	The values to which coefficients are fixed.
<code>model\$coef_fix_other</code>	The multiples of the estimated coefficients, which are added to the fixed coefficients.
<code>model\$coef_fix_special</code>	The predefined structures of <code>coef_fix_value</code> and <code>coef_fix_other</code> .
<code>model\$coef_bound_lower</code>	The lower bounds on coefficients.
<code>model\$coef_bound_upper</code>	The upper bounds on coefficients.
<code>model\$coef_est</code>	The estimated coefficients.
<code>bootstrap\$method</code>	The method used for bootstrapping.
<code>bootstrap\$coef_set</code>	The bootstrapped sets of coefficients.

```
bootstrap$coef_mean
    The mean of bootstrapped coefficients.
bootstrap$coef_vcov
    The variance-covariance matrix of bootstrapped coefficients.
bootstrap$coef_sd
    The standard deviation of bootstrapped coefficients.
bootstrap$coef_quant
    The quantiles of bootstrapped coefficients.
```

References

Creal, D., Koopman, S. J., and Lucas, A. (2013). Generalized Autoregressive Score Models with Applications. *Journal of Applied Econometrics*, **28**(5), 777–795. doi: [10.1002/jae.1279](https://doi.org/10.1002/jae.1279).

Harvey, A. C. (2013). *Dynamic Models for Volatility and Heavy Tails: With Applications to Financial and Economic Time Series*. Cambridge University Press. doi: [10.1017/cbo9781139540933](https://doi.org/10.1017/cbo9781139540933).

See Also

[gas\(\)](#)

Examples

```
# Load Level of Lake Huron dataset
data(LakeHuron)
y <- LakeHuron - 570
x <- 1:length(y)

# Estimate GAS model based on the normal distribution with dynamic mean
est_gas <- gas(y = y, x = x, distr = "norm", spec = "reg_err",
  par_static = c(FALSE, TRUE), coef_start = c(9.99, -0.02, 0.46, 0.67, 0.46))
est_gas

# Bootstrap the model (can be time-consuming for a larger number of samples)
boot_gas <- gas_bootstrap(est_gas, rep_boot = 10)
boot_gas
```

gas_filter

Filter GAS Model

Description

A function for obtaining filtered time-varying parameters of generalized autoregressive score (GAS) models of Creal et al. (2013) and Harvey (2013). It captures parameter uncertainty and can also be used for forecasting. Method "simulated_coefs" computes a path of time-varying parameters for each simulated coefficient set under assumption of asymptotic normality with given variance-covariance matrix (see Blasques et al., 2016). Method "given_coefs" computes a path of time-varying parameters for each supplied coefficient set. Instead of supplying arguments about the model, the function can be applied to the gas object obtained by the [gas\(\)](#) function.

Usage

```
gas_filter(gas_object = NULL, method = "simulated_coefs",
  coef_set = NULL, rep_gen = 1000L, t_ahead = 0L, x_ahead = NULL,
  rep_ahead = 1000L, quant = c(0.025, 0.975), y = NULL, x = NULL,
  distr = NULL, param = NULL, scaling = "unit", spec = "joint",
  p = 1L, q = 1L, par_static = NULL, par_link = NULL,
  par_init = NULL, coef_fix_value = NULL, coef_fix_other = NULL,
  coef_fix_special = NULL, coef_bound_lower = NULL,
  coef_bound_upper = NULL, coef_est = NULL, coef_vcov = NULL)
```

Arguments

gas_object	An optional GAS estimate, i.e. a list of S3 class gas returned by function gas() .
method	A method used for parameter uncertainty. Supported methods are "given_coefs" and "simulated_coefs".
coef_set	A numeric matrix of coefficient sets in rows for method = "given_coefs". Can be generated for example by gas_bootstrap() .
rep_gen	A number of generated coefficient sets for method = "simulated_coefs".
t_ahead	A number of observations to forecast.
x_ahead	Out-of-sample exogenous variables. For a single variable common for all time-varying parameters, a numeric vector. For multiple variables common for all time-varying parameters, a numeric matrix with observations in rows. For individual variables for each time-varying parameter, a list of numeric vectors or matrices in the above form. The number of observation must be equal to t_ahead.
rep_ahead	A number of simulation repetitions for forecasting when t_ahead > 0.
quant	A numeric vector of probabilities determining quantiles.
y	A time series. For univariate time series, a numeric vector or a matrix with a single column. For multivariate times series, a numeric matrix with observations in rows.
x	Optional exogenous variables. For a single variable common for all time-varying parameters, a numeric vector. For multiple variables common for all time-varying parameters, a numeric matrix with observations in rows. For individual variables for each time-varying parameter, a list of numeric vectors or matrices in the above form. The number of observation must be equal to the number of observations of y.
distr	A conditional distribution. See distr() for available distributions.
param	A parametrization of the conditional distribution. If NULL, default parametrization is used. See distr() for available parametrizations.
scaling	A scaling function for the score. The supported scaling functions are the unit scaling (scaling = "unit"), the inverse of the Fisher information matrix scaling (scaling = "fisher_inv"), and the inverse square root of the Fisher information matrix scaling (scaling = "fisher_inv_sqrt"). The latter two scalings use the Fisher information for the time-varying parameters only. For the full Fisher information matrix for both time-varying and static parameters, there are

the "full_fisher_inv" and "full_fisher_inv_sqrt" scalings. For the individual Fisher information for each parameter, there are the "diag_fisher_inv" and "diag_fisher_inv_sqrt" scalings. Note that when the parametrization is orthogonal (see `distr()`), there are no differences between these scaling variants.

spec	A specification of the dynamic equation with regard to exogenous variables. The supported specifications are exogenous variables and dynamics within the same equation (spec = "joint") and separate equations for exogenous variables and dynamics in the fashion of regression models with dynamic errors (spec = "reg_err"). In a stationary model without exogenous variables, the two specifications are equivalent, although with differently parametrized intercept.
p	A score order. For order common for all parameters, a numeric vector of length 1. For individual order for each parameter, a numeric vector of length equal to the number of parameters. Defaults to 1L.
q	An autoregressive order. For order common for all parameters, a numeric vector of length 1. For individual order for each parameter, a numeric vector of length equal to the number of parameters. Defaults to 1L.
par_static	An optional logical vector indicating static parameters. Overrides x, p, and q.
par_link	An optional logical vector indicating whether the logarithmic/logistic link should be applied to restricted parameters in order to obtain unrestricted values. Defaults to applying the logarithmic/logistic link for time-varying parameters and keeping the original link for constant parameters.
par_init	An optional numeric vector of initial values of time-varying parameters. For NA values or when NULL, set initial values to unconditional values of time-varying parameters. For example, in the case of GAS(1,1) model with spec = "joint", to $\omega / (1 - \phi_1)$. Not to be confused with starting values for the optimization <code>coef_start</code> .
coef_fix_value	An optional numeric vector of values to which coefficients are to be fixed. NA values represent coefficients to be estimated.
coef_fix_other	An optional square numeric matrix of multiples of the estimated coefficients, which are to be added to the fixed coefficients. This allows the fixed coefficients to be linear combinations of the estimated coefficients. A coefficient given by row is fixed on coefficient given by column. By this logic, all rows corresponding to the estimated coefficients should contain only NA values. Furthermore, all columns corresponding to the fixed coefficients should also contain only NA values.
coef_fix_special	An optional character vector of predefined structures of <code>coef_fix_value</code> and <code>coef_fix_other</code> . Useful mainly for multidimensional models. Value "panel_structure" forces all regression, autoregression, and score coefficients to be the same for all time-varying parameters within their group. Value "zero_sum_intercept" forces all constant parameters to sum up to zero within their group. Value "random_walk" forces all autoregressive coefficients to be equal to one (should be used with caution due to nonstationarity; <code>par_init</code> must be specified). Multiple predefined structures can be used together. Also can be used in combination with custom <code>coef_fix_value</code> and <code>coef_fix_other</code> .

coef_bound_lower	An optional numeric vector of lower bounds on coefficients.
coef_bound_upper	An optional numeric vector of upper bounds on coefficients.
coef_est	A numeric vector of estimated coefficients.
coef_vcov	A numeric matrix of estimated covariances between coefficients.

Value

A list with components:

data\$y	The time series.
data\$x	The exogenous variables.
data\$x_ahead	The out-of-sample exogenous variables. Only when $t_ahead > 0$.
model\$distr	The conditional distribution.
model\$param	The parametrization of the conditional distribution.
model\$scaling	The scaling function.
model\$spec	The specification of the dynamic equation.
model\$t	The length of the time series.
model\$t_ahead	The length of the out-of-sample time series. Only when $t_ahead > 0$.
model\$n	The dimension of the model.
model\$m	The number of exogenous variables.
model\$p	The score order.
model\$q	The autoregressive order.
model\$par_static	The static parameters.
model\$par_link	The parameters with the logarithmic/logistic links.
model\$par_init	The initial values of the time-varying parameters.
model\$coef_fix_value	The values to which coefficients are fixed.
model\$coef_fix_other	The multiples of the estimated coefficients, which are added to the fixed coefficients.
model\$coef_fix_special	The predefined structures of <code>coef_fix_value</code> and <code>coef_fix_other</code> .
model\$coef_bound_lower	The lower bounds on coefficients.
model\$coef_bound_upper	The upper bounds on coefficients.
model\$coef_set	The coefficient sets.
filter\$method	The method used for parameter uncertainty.
filter\$par_tv_mean	The mean of the time-varying parameters.

filter\$par_tv_sd	The standard deviation of the time-varying parameters.
filter\$par_tv_quant	The quantiles of the time-varying parameters.
filter\$score_tv_mean	The mean of the scores.
filter\$score_tv_sd	The standard deviation of the scores.
filter\$score_tv_quant	The quantiles of the scores.
filter\$y_ahead_mean	The mean of the forecasted time series. Only when $t_ahead > 0$.
filter\$y_ahead_sd	The standard deviation of the forecasted time series. Only when $t_ahead > 0$.
filter\$y_ahead_quant	The quantiles of the forecasted time series. Only when $t_ahead > 0$.
filter\$par_tv_ahead_mean	The mean of the forecasted time-varying parameters. Only when $t_ahead > 0$.
filter\$par_tv_ahead_sd	The standard deviation of the forecasted time-varying parameters. Only when $t_ahead > 0$.
filter\$par_tv_ahead_quant	The quantiles of the forecasted time-varying parameters. Only when $t_ahead > 0$.
filter\$score_tv_ahead_mean	The mean of the forecasted scores. Only when $t_ahead > 0$.
filter\$score_tv_ahead_sd	The standard deviation of the forecasted scores. Only when $t_ahead > 0$.
filter\$score_tv_ahead_quant	The quantiles of the forecasted scores. Only when $t_ahead > 0$.

References

- Blasques, F., Koopman, S. J., Łasak, K., and Lucas, A. (2016). In-Sample Confidence Bands and Out-of-Sample Forecast Bands for Time-Varying Parameters in Observation-Driven Models. *International Journal of Forecasting*, **32**(3), 875–887. doi: [10.1016/j.ijforecast.2015.11.018](https://doi.org/10.1016/j.ijforecast.2015.11.018).
- Creal, D., Koopman, S. J., and Lucas, A. (2013). Generalized Autoregressive Score Models with Applications. *Journal of Applied Econometrics*, **28**(5), 777–795. doi: [10.1002/jae.1279](https://doi.org/10.1002/jae.1279).
- Harvey, A. C. (2013). *Dynamic Models for Volatility and Heavy Tails: With Applications to Financial and Economic Time Series*. Cambridge University Press. doi: [10.1017/cbo9781139540933](https://doi.org/10.1017/cbo9781139540933).

See Also

[gas\(\)](#)

Examples

```
# Load Level of Lake Huron dataset
data(LakeHuron)
y <- LakeHuron - 570
x <- 1:length(y)

# Estimate GAS model based on the normal distribution with dynamic mean
est_gas <- gas(y = y, x = x, distr = "norm", spec = "reg_err",
  par_static = c(FALSE, TRUE), coef_start = c(9.99, -0.02, 0.46, 0.67, 0.46))
est_gas

# Filter the time-varying parameters by the "simulated_coefs" method
flt_gas <- gas_filter(est_gas, rep_gen = 100)
flt_gas
```

gas_forecast

*Forecast GAS Model***Description**

A function for forecasting of generalized autoregressive score (GAS) models of Creal et al. (2013) and Harvey (2013). Method "mean_path" filters time-varying parameters based on zero score and then generates mean of time series. Method "simulated_paths" repeatedly simulates time series, simultaneously filters time-varying parameters, and then estimates mean, standard deviation, and quantiles (see Blasques et al., 2016). Instead of supplying arguments about the model, the function can be applied to the gas object obtained by the `gas()` function.

Usage

```
gas_forecast(gas_object = NULL, method = "mean_path", t_ahead = 1L,
  x_ahead = NULL, rep_ahead = 1000L, quant = c(0.025, 0.975), y = NULL,
  x = NULL, distr = NULL, param = NULL, scaling = "unit",
  spec = "joint", p = 1L, q = 1L, par_static = NULL, par_link = NULL,
  par_init = NULL, coef_est = NULL)
```

Arguments

gas_object	An optional GAS estimate, i.e. a list of S3 class gas returned by function <code>gas()</code> .
method	A method used for forecasting. Supported methods are "mean_path" and "simulated_paths".
t_ahead	A number of observations to forecast.
x_ahead	Out-of-sample exogenous variables. For a single variable common for all time-varying parameters, a numeric vector. For multiple variables common for all time-varying parameters, a numeric matrix with observations in rows. For individual variables for each time-varying parameter, a list of numeric vectors or matrices in the above form. The number of observation must be equal to t_ahead.

rep Ahead	A number of simulation repetitions for method = "simulated_paths".
quant	A numeric vector of probabilities determining quantiles for method = "simulated_paths".
y	A time series. For univariate time series, a numeric vector or a matrix with a single column. For multivariate times series, a numeric matrix with observations in rows.
x	Optional exogenous variables. For a single variable common for all time-varying parameters, a numeric vector. For multiple variables common for all time-varying parameters, a numeric matrix with observations in rows. For individual variables for each time-varying parameter, a list of numeric vectors or matrices in the above form. The number of observation must be equal to the number of observations of y.
distr	A conditional distribution. See <code>distr()</code> for available distributions.
param	A parametrization of the conditional distribution. If NULL, default parametrization is used. See <code>distr()</code> for available parametrizations.
scaling	A scaling function for the score. The supported scaling functions are the unit scaling (scaling = "unit"), the inverse of the Fisher information matrix scaling (scaling = "fisher_inv"), and the inverse square root of the Fisher information matrix scaling (scaling = "fisher_inv_sqrt"). The latter two scalings use the Fisher information for the time-varying parameters only. For the full Fisher information matrix for both time-varying and static parameters, there are the "full_fisher_inv" and "full_fisher_inv_sqrt" scalings. For the individual Fisher information for each parameter, there are the "diag_fisher_inv" and "diag_fisher_inv_sqrt" scalings. Note that when the parametrization is orthogonal (see <code>distr()</code>), there are no differences between these scaling variants.
spec	A specification of the dynamic equation with regard to exogeneous variables. The supported specifications are exogenous variables and dynamics within the same equation (spec = "joint") and separate equations for exogenous variables and dynamics in the fashion of regression models with dynamic errors (spec = "reg_err"). In a stationary model without exogenous variables, the two specifications are equivalent, although with differently parametrized intercept.
p	A score order. For order common for all parameters, a numeric vector of length 1. For individual order for each parameter, a numeric vector of length equal to the number of parameters. Defaults to 1L.
q	An autoregressive order. For order common for all parameters, a numeric vector of length 1. For individual order for each parameter, a numeric vector of length equal to the number of parameters. Defaults to 1L.
par_static	An optional logical vector indicating static parameters. Overrides x, p, and q.
par_link	An optional logical vector indicating whether the logarithmic/logistic link should be applied to restricted parameters in order to obtain unrestricted values. Defaults to applying the logarithmic/logistic link for time-varying parameters and keeping the original link for constant parameters.
par_init	An optional numeric vector of initial values of time-varying parameters. For NA values or when NULL, set initial values to unconditional values of time-varying parameters. For example, in the case of GAS(1,1) model with spec = "joint",

to $\omega / (1 - \phi_1)$. Not to be confused with starting values for the optimization `coef_start`.

`coef_est` A numeric vector of estimated coefficients.

Value

A list with components:

`data$y` The time series.

`data$x` The exogenous variables.

`data$x_ahead` The out-of-sample exogenous variables.

`model$distr` The conditional distribution.

`model$param` The parametrization of the conditional distribution.

`model$scaling` The scaling function.

`model$spec` The specification of the dynamic equation.

`model$t` The length of the time series.

`model$t_ahead` The length of the out-of-sample time series.

`model$n` The dimension of the model.

`model$m` The number of exogenous variables.

`model$p` The score order.

`model$q` The autoregressive order.

`model$par_static`
The static parameters.

`model$par_link` The parameters with the logarithmic/logistic links.

`model$par_init` The initial values of the time-varying parameters.

`model$coef_est` The estimated coefficients.

`forecast$method`
The method used for forecasting.

`forecast$y_ahead_mean`
The mean of the forecasted time series.

`forecast$y_ahead_sd`
The standard deviation of the forecasted time series. Only for `method = "simulated_paths"`.

`forecast$y_ahead_quant`
The quantiles of the forecasted time series. Only for `method = "simulated_paths"`.

`forecast$par_tv_ahead_mean`
The mean of the forecasted time-varying parameters.

`forecast$par_tv_ahead_sd`
The standard deviation of the forecasted time-varying parameters. Only for `method = "simulated_paths"`.

`forecast$par_tv_ahead_quant`
The quantiles of the forecasted time-varying parameters. Only for `method = "simulated_paths"`.

forecast\$score_tv_ahead_mean
The mean of the forecasted scores.

forecast\$score_tv_ahead_sd
The standard deviation of the forecasted scores. Only for method = "simulated_paths".

forecast\$score_tv_ahead_quant
The quantiles of the forecasted scores. Only for method = "simulated_paths".

References

Blasques, F., Koopman, S. J., Łasak, K., and Lucas, A. (2016). In-Sample Confidence Bands and Out-of-Sample Forecast Bands for Time-Varying Parameters in Observation-Driven Models. *International Journal of Forecasting*, **32**(3), 875–887. doi: [10.1016/j.ijforecast.2015.11.018](https://doi.org/10.1016/j.ijforecast.2015.11.018).

Creal, D., Koopman, S. J., and Lucas, A. (2013). Generalized Autoregressive Score Models with Applications. *Journal of Applied Econometrics*, **28**(5), 777–795. doi: [10.1002/jae.1279](https://doi.org/10.1002/jae.1279).

Harvey, A. C. (2013). *Dynamic Models for Volatility and Heavy Tails: With Applications to Financial and Economic Time Series*. Cambridge University Press. doi: [10.1017/cbo9781139540933](https://doi.org/10.1017/cbo9781139540933).

See Also

[gas\(\)](#)

Examples

```
# Load Level of Lake Huron dataset
data(LakeHuron)
y <- LakeHuron - 570
x <- 1:length(y)

# Estimate GAS model based on the normal distribution with dynamic mean
est_gas <- gas(y = y, x = x, distr = "norm", spec = "reg_err",
  par_static = c(FALSE, TRUE), coef_start = c(9.99, -0.02, 0.46, 0.67, 0.46))
est_gas

# Forecast the model by the "simulated_paths" method
fcst_gas <- gas_forecast(est_gas, method = "simulated_paths",
  t_ahead = 22, x_ahead = 99:120, rep_ahead = 100)
fcst_gas

# Plot the forecasted expected value with the confidence interval
plot(c(fcst_gas$data$y, fcst_gas$forecast$y_ahead_mean), type = "b")
lines(99:120, fcst_gas$forecast$y_ahead_quant[, 1], col = "blue")
lines(99:120, fcst_gas$forecast$y_ahead_quant[, 2], col = "blue")
```

gas_simulate

*Simulate GAS Model***Description**

A function for simulation of generalized autoregressive score (GAS) models of Creal et al. (2013) and Harvey (2013). Instead of supplying arguments about the model, the function can be applied to the gas object obtained by the `gas()` function.

Usage

```
gas_simulate(gas_object = NULL, t_sim = 1L, x_sim = NULL, distr = NULL,
             param = NULL, scaling = "unit", spec = "joint", n = NULL, p = 1L,
             q = 1L, par_static = NULL, par_link = NULL, par_init = NULL,
             coef_est = NULL)
```

Arguments

gas_object	An optional GAS estimate, i.e. a list of S3 class gas returned by function <code>gas()</code> .
t_sim	A number of observations to simulate.
x_sim	Exogenous variables used for simulations. For a single variable common for all time-varying parameters, a numeric vector. For multiple variables common for all time-varying parameters, a numeric matrix with observations in rows. For individual variables for each time-varying parameter, a list of numeric vectors or matrices in the above form. The number of observation must be equal to t_sim.
distr	A conditional distribution. See <code>distr()</code> for available distributions.
param	A parametrization of the conditional distribution. If NULL, default parametrization is used. See <code>distr()</code> for available parametrizations.
scaling	A scaling function for the score. The supported scaling functions are the unit scaling (scaling = "unit"), the inverse of the Fisher information matrix scaling (scaling = "fisher_inv"), and the inverse square root of the Fisher information matrix scaling (scaling = "fisher_inv_sqrt"). The latter two scalings use the Fisher information for the time-varying parameters only. For the full Fisher information matrix for both time-varying and static parameters, there are the "full_fisher_inv" and "full_fisher_inv_sqrt" scalings. For the individual Fisher information for each parameter, there are the "diag_fisher_inv" and "diag_fisher_inv_sqrt" scalings. Note that when the parametrization is orthogonal (see <code>distr()</code>), there are no differences between these scaling variants.
spec	A specification of the dynamic equation with regard to exogeneous variables. The supported specifications are exogenous variables and dynamics within the same equation (spec = "joint") and separate equations for exogenous variables and dynamics in the fashion of regression models with dynamic errors (spec = "reg_err"). In a stationary model without exogenous variables, the two specifications are equivalent, although with differently parametrized intercept.

n	A dimension of the model. Required only for multivariate models.
p	A score order. For order common for all parameters, a numeric vector of length 1. For individual order for each parameter, a numeric vector of length equal to the number of parameters. Defaults to 1L.
q	An autoregressive order. For order common for all parameters, a numeric vector of length 1. For individual order for each parameter, a numeric vector of length equal to the number of parameters. Defaults to 1L.
par_static	An optional logical vector indicating static parameters. Overrides x, p, and q.
par_link	An optional logical vector indicating whether the logarithmic/logistic link should be applied to restricted parameters in order to obtain unrestricted values. Defaults to applying the logarithmic/logistic link for time-varying parameters and keeping the original link for constant parameters.
par_init	An optional numeric vector of initial values of time-varying parameters. For NA values or when NULL, set initial values to unconditional values of time-varying parameters. For example, in the case of GAS(1,1) model with spec = "joint", to $\omega / (1 - \phi_1)$. Not to be confused with starting values for the optimization coef_start.
coef_est	A numeric vector of estimated coefficients.

Value

A list with components:

data\$x_sim	The exogenous variables used in simulation.
model\$distr	The conditional distribution.
model\$param	The parametrization of the conditional distribution.
model\$scaling	The scaling function.
model\$spec	The specification of the dynamic equation.
model\$t_sim	The length of the simulated time series.
model\$n	The dimension of the model.
model\$m	The number of exogenous variables.
model\$p	The score order.
model\$q	The autoregressive order.
model\$par_static	The static parameters.
model\$par_link	The parameters with the logarithmic/logistic links.
model\$par_init	The initial values of the time-varying parameters.
model\$coef_est	The estimated coefficients.
simulation\$y_sim	The simulated time series.
simulation\$par_tv_sim	The simulated time-varying parameters.
simulation\$score_tv_sim	The simulated scores.

References

Creal, D., Koopman, S. J., and Lucas, A. (2013). Generalized Autoregressive Score Models with Applications. *Journal of Applied Econometrics*, **28**(5), 777–795. doi: [10.1002/jae.1279](https://doi.org/10.1002/jae.1279).

Harvey, A. C. (2013). *Dynamic Models for Volatility and Heavy Tails: With Applications to Financial and Economic Time Series*. Cambridge University Press. doi: [10.1017/cbo9781139540933](https://doi.org/10.1017/cbo9781139540933).

See Also

[gas\(\)](#)

Examples

```
# Simulate GAS model based on the Weibull distribution with dynamic scale
sim_gas <- gas_simulate(t_sim = 50, distr = "weibull",
  par_static = c(FALSE, TRUE), coef_est = c(0.2, 0.1, 0.8, 2.0))
sim_gas

# Plot the simulated time series
plot(sim_gas$simulation$y_sim, type = "b")
```

german_car_market_cap *Market Capitalization of German Car Manufacturers*

Description

Market capitalization of the "Germany's Big Three" automobile manufacturers – Volkswagen Group, Mercedes-Benz Group, and BMW. Market capitalization is reported in billions of euros and covers the period 1994–2021.

Usage

```
german_car_market_cap
```

Format

A data frame with columns:

year Year.

car_manufacturer Car manufacturer.

market_cap Market capitalization in billions of euros.

Source

Thomson Reuters (www.thomsonreuters.com).

ice_hockey_championships

Results of the Ice Hockey World Championships

Description

The dataset contains the results of the annual men's Ice Hockey World Championships from 1998 to 2021. In 1998, the International Ice Hockey Federation set the number of teams participating in the championships at 16. Since 1998, a total of 24 teams have qualified for the championship division. This dataset is analyzed in Holý and Zouhar (2022).

Usage

ice_hockey_championships

Format

A list with components:

rankings A matrix of final rankings. Rows correspond to years, columns to teams. Value Inf means that the team did not advance to the championship. Value NA means that the championship did not take place.

hosts A matrix of dummy variables indicating whether the team hosted the championship. Rows correspond to years, columns to teams. Multiple hosts of one championship is possible. Value NA means that the championship did not take place.

Source

International Ice Hockey Federation (www.iihf.com).

References

Holý, V. and Zouhar, J. (2022). Modelling Time-Varying Rankings with Autoregressive and Score-Driven Dynamics. *Journal of the Royal Statistical Society: Series C (Applied Statistics)*, **71**(5). doi: [10.1111/rssc.12584](https://doi.org/10.1111/rssc.12584).

sp500_daily

Daily S&P 500 Prices

Description

Daily opening, highest, lowest, and closing prices of the Standard and Poor's 500 stock market index (SPX) from 2013.

Usage

```
sp500_daily
```

Format

A data frame with columns:

date Trading day.

open Opening price of the day.

high Highest price of the day.

low Lowest price of the day.

close Closing price of the day.

Source

Nasdaq (www.nasdaq.com/market-activity/index/spx).

wrappers_hessian	<i>Wrappers for Hessian Functions</i>
------------------	---------------------------------------

Description

Wrappers of common R Hessian functions. Their purpose is to be passed as the `hessian_function` argument in the `gas()` function.

Usage

```
wrapper_hessian_stats(obj_fun, theta_optim, ...)
```

```
wrapper_hessian_pracma(obj_fun, theta_optim, ...)
```

```
wrapper_hessian_numderiv(obj_fun, theta_optim, ...)
```

Arguments

`obj_fun` An objective function.

`theta_optim` A numeric vector of the optimal values of the variables.

`...` Additional arguments to be passed to the Hessian function. These include arguments required by the objective function, namely `est_details` and `print_progress`.

Value

A list with components:

`status_hessian` The status of the Hessian computation.

`theta_hessian` The Hessian matrix.

Functions

- `wrapper_hessian_stats()`: Wrapper for Hessian function `stats::optimHess()`.
- `wrapper_hessian_pracma()`: Wrapper for Hessian function `pracma::hessian()`.
- `wrapper_hessian_numderiv()`: Wrapper for Hessian function `numDeriv::hessian()`.

See Also

`gas()` `wrappers_optim`

wrappers_optim

Wrappers for Optimization Functions

Description

Wrappers of common R optimization functions. Their purpose is to be passed as the `optim_function` argument in the `gas()` function.

Usage

```
wrapper_optim_stats(obj_fun, theta_start, theta_bound_lower, theta_bound_upper,
  ...)
```

```
wrapper_optim_nloptr(obj_fun, theta_start, theta_bound_lower,
  theta_bound_upper, ...)
```

Arguments

<code>obj_fun</code>	An objective function.
<code>theta_start</code>	A numeric vector of starting values of the variables.
<code>theta_bound_lower</code>	A numeric vector of lower bounds on the variables.
<code>theta_bound_upper</code>	A numeric vector of upper bounds on the variables.
<code>...</code>	Additional arguments to be passed to the optimization function. These include arguments required by the objective function, namely <code>est_details</code> and <code>print_progress</code> .

Value

A list with components:

<code>status_optim</code>	The status of the optimization computation.
<code>theta_optim</code>	The optimal solution.

Functions

- `wrapper_optim_stats()`: Wrapper for optimization function `stats::optim()`.
- `wrapper_optim_nloptr()`: Wrapper for optimization function `nloptr::nloptr()`.

See Also[gas\(\)](#) [wrappers_hessian](#)

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