# Package 'greta.dynamics'

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Type Package

Title Modelling Structured Dynamical Systems in 'greta'

Version 0.2.0

**Description** A 'greta' extension for analysing transition matrices and ordinary differential equations representing dynamical systems. Provides functions for analysing transition matrices by iteration, and solving ordinary differential equations. This is an extension to the 'greta' software, Golding (2019) <doi:10.21105/joss.01601>.

**License** Apache License (>= 2)

URL https://github.com/greta-dev/greta.dynamics,

https://greta-dev.github.io/greta.dynamics/

BugReports https://github.com/greta-dev/greta.dynamics/issues

**Imports** cli, glue, tensorflow (>= 1.14.0)

**Depends** greta (>= 0.4.2), R (>= 3.1.0)

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SystemRequirements Python (>= 2.7.0) with header files and shared library; TensorFlow (v1.14; https://www.tensorflow.org/); TensorFlow Probability (v0.7.0; https://www.tensorflow.org/probability/)

NeedsCompilation no

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greta.dynamics greta.dynamics: a greta extension for modelling dynamical systems

#### Description

an extension to greta with functions for simulating dynamical systems, defined by of ordinary differential equations (see ode\_solve()) or transition matrices (iterate\_matrix()).

iterate\_matrix *iterate transition matrices* 

#### Description

Calculate the intrinsic growth rate(s) and stable stage distribution(s) for a stage-structured dynamical system, encoded as state\_t = matrix % state\_tm1.

#### Usage

```
iterate_matrix(
  matrix,
  initial_state = rep(1, ncol(matrix)),
  niter = 100,
  tol = 1e-06
)
```

#### Arguments

matrix	either a square 2D transition matrix (with dimensions m x m), or a 3D array (with dimensions n x m x m), giving one or more transition matrices to iterate
initial_state	either a column vector (with m elements) or a 3D array (with dimensions n x m x 1) giving one or more initial states from which to iterate the matrix
niter	a positive integer giving the maximum number of times to iterate the matrix
tol	a scalar giving a numerical tolerance, below which the algorithm is determined to have converged to the same growth rate in all stages

#### iterate\_matrix

#### Details

iterate\_matrix can either act on a single transition matrix and initial state (if matrix is 2D and initial\_state is a column vector), or it can simultaneously act on n different matrices and/or n different initial states (if matrix and initial\_state are 3D arrays). In the latter case, the first dimension of both objects should be the batch dimension n.

To ensure the matrix is iterated for a specific number of iterations, you can set that number as niter, and set tol to 0 or a negative number to ensure that the iterations are not stopped early.

#### Value

a named list with five greta arrays:

- lambda a scalar or vector giving the ratio of the first stage values between the final two iterations.
- stable\_state a vector or matrix (with the same dimensions as initial\_state) giving the state after the final iteration, normalised so that the values for all stages sum to one.
- all\_states an n x m x niter matrix of the state values at each iteration. This will be 0 for all entries after iterations.
- converged an integer scalar or vector indicating whether the iterations for each matrix have converged to a tolerance less than tol (1 if so, 0 if not) before the algorithm finished.
- iterations a scalar of the maximum number of iterations completed before the algorithm terminated. This should match niter if converged is FALSE.

#### Note

because greta vectorises across both MCMC chains and the calculation of greta array values, the algorithm is run until all chains (or posterior samples), sites and stages have converged to stable growth. So a single value of both converged and iterations is returned, and the value of this will always have the same value in an mcmc.list object. So inspecting the MCMC trace of these parameters will only tell you whether the iteration converged in *all* posterior samples, and the maximum number of iterations required to do so across all these samples

#### Examples

```
## Not run:
# simulate from a probabilistic 4-stage transition matrix model
k <- 4
# component variables
# survival probability for all stages
survival <- uniform(0, 1, dim = k)
# conditional (on survival) probability of staying in a stage
stasis <- c(uniform(0, 1, dim = k - 1), 1)
# marginal probability of staying/progressing
stay <- survival * stasis
progress <- (survival * (1 - stay))[1:(k - 1)]
# recruitment rate for the largest two stages
recruit <- exponential(c(3, 5))</pre>
```

```
# combine into a matrix:
tmat <- zeros(k, k)</pre>
diag(tmat) <- stay</pre>
progress_idx <- row(tmat) - col(tmat) == 1</pre>
tmat[progress_idx] <- progress</pre>
tmat[1, k - (1:0)] <- recruit</pre>
# analyse this to get the intrinsic growth rate and stable state
iterations <- iterate_matrix(tmat)</pre>
iterations$lambda
iterations$stable_distribution
iterations$all_states
# Can also do this simultaneously for a collection of transition matrices
k <- 2
n <- 10
survival <- uniform(0, 1, dim = c(n, k))</pre>
stasis <- cbind(uniform(0, 1, dim = n), rep(1, n))</pre>
stay <- survival * stasis</pre>
progress <- (survival * (1 - stasis))[, 1]</pre>
recruit_rate <- 1 / seq(0.1, 5, length.out = n)</pre>
recruit <- exponential(recruit_rate, dim = n)</pre>
tmats <- zeros(10, 2, 2)</pre>
tmats[, 1, 1] <- stasis[, 1]</pre>
tmats[, 2, 2] <- stasis[, 2]</pre>
tmats[, 2, 1] <- progress</pre>
tmats[, 1, 2] <- recruit</pre>
iterations <- iterate_matrix(tmats)</pre>
iterations$lambda
iterations$stable_distribution
iterations$all_states
## End(Not run)
```

solve ODEs

#### Description

ode\_solve

Solve a system of ordinary differential equations.

#### Usage

```
ode_solve(derivative, y0, times, ..., method = c("ode45", "rk4", "midpoint"))
```

#### Arguments

derivative a derivative function. The first two arguments must be 'y' and 't', the state parameter and scalar timestep respectively. The remaining parameters must be

```
4
```

	named arguments representing (temporally static) model parameters. Variables and distributions cannot be defined in the function.
y0	a greta array for the value of the state parameter y at time 0
times	a column vector of times at which to evaluate y
	named arguments giving greta arrays for the additional (fixed) parameters
method	which solver to use. "ode45" uses adaptive step sizes, whilst "rk4" and "midpoint" use the fixed grid defined by times; they may be faster but less accurate than "ode45".

#### Value

greta array

#### Examples

```
## Not run:
# replicate the Lotka-Volterra example from deSolve
library(deSolve)
LVmod <- function(Time, State, Pars) {</pre>
  with(as.list(c(State, Pars)), {
    Ingestion <- rIng * Prey * Predator</pre>
    GrowthPrey <- rGrow * Prey * (1 - Prey / K)</pre>
    MortPredator <- rMort * Predator</pre>
    dPrey <- GrowthPrey - Ingestion
    dPredator <- Ingestion * assEff - MortPredator
    return(list(c(dPrey, dPredator)))
 })
}
pars <- c(
  rIng = 0.2, # /day, rate of ingestion
  rGrow = 1.0, # /day, growth rate of prey
  rMort = 0.2, # /day, mortality rate of predator
  assEff = 0.5, # -, assimilation efficiency
  K = 10
) # mmol/m3, carrying capacity
yini <- c(Prey = 1, Predator = 2)</pre>
times <- seq(0, 30, by = 1)
out <- ode(yini, times, LVmod, pars)</pre>
# simulate observations
jitter <- rnorm(2 * length(times), 0, 0.1)</pre>
y_obs <- out[, -1] + matrix(jitter, ncol = 2)</pre>
# ~~~~~~~
# fit a greta model to infer the parameters from this simulated data
# greta version of the function
```

```
lotka_volterra <- function(y, t, rIng, rGrow, rMort, assEff, K) {</pre>
  Prey <- y[1, 1]
  Predator <- y[1, 2]
  Ingestion <- rIng * Prey * Predator</pre>
  GrowthPrey <- rGrow * Prey * (1 - Prey / K)</pre>
  MortPredator <- rMort * Predator</pre>
  dPrey <- GrowthPrey - Ingestion
  dPredator <- Ingestion * assEff - MortPredator
  cbind(dPrey, dPredator)
}
# priors for the parameters
rIng <- uniform(0, 2) # /day, rate of ingestion</pre>
rGrow <- uniform(0, 3) # /day, growth rate of prey</pre>
rMort <- uniform(0, 1) # /day, mortality rate of predator</pre>
assEff <- uniform(0, 1) # -, assimilation efficiency</pre>
K <- uniform(0, 30) # mmol/m3, carrying capacity</pre>
# initial values and observation error
y0 <- uniform(0, 5, dim = c(1, 2))
obs_sd <- uniform(0, 1)</pre>
# solution to the ODE
y <- ode_solve(lotka_volterra, y0, times, rIng, rGrow, rMort, assEff, K)</pre>
# sampling statement/observation model
distribution(y_obs) <- normal(y, obs_sd)</pre>
# we can use greta to solve directly, for a fixed set of parameters (the true
# ones in this case)
values <- c(</pre>
  list(y0 = t(1:2)),
  as.list(pars)
)
vals <- calculate(y, values = values)[[1]]</pre>
plot(vals[, 1] ~ times, type = "1", ylim = range(vals))
lines(vals[, 2] ~ times, lty = 2)
points(y_obs[, 1] ~ times)
points(y_obs[, 2] ~ times, pch = 2)
# or we can do inference on the parameters:
# build the model (takes a few seconds to define the tensorflow graph)
m <- model(rIng, rGrow, rMort, assEff, K, obs_sd)</pre>
# compute MAP estimate
o <- opt(m)</pre>
0
## End(Not run)
```

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