Package ‘wstdiff’

November 11, 2025

Version 1.0.0

Title Welch-Satterthwaite Approximation for t-Distribution Differences

Description Implements the Welch-Satterthwaite approximation for differences of non-standardized t-distributed random variables in both univariate and multivariate settings. The package provides methods for computing effective degrees of freedom and scale parameters, as well as distribution functions for the approximated difference distribution. The methodology extends the classical Welch-Satterthwaite framework from variance combinations to t-distribution differences through careful moment matching. Methods build on the classical Welch-Satterthwaite approach described in Welch (1947) <doi:10.1093/biomet/34.1-2.28> and Satterthwaite (1946) <doi:10.2307/3002019>.

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Encoding UTF-8

RoxygenNote 7.3.3

Depends R (>= 3.5.0)

Imports stats

Suggests testthat (>= 3.0.0), mvtnorm, knitr, rmarkdown

VignetteBuilder knitr

NeedsCompilation no

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Repository CRAN

Date/Publication 2025-11-11 21:40:02 UTC

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tdiff_distributions *Distribution Functions for Approximated t-Difference*

Description

Distribution Functions for Approximated t-Difference

Usage

```
dtdiff(x, ws_result)

ptdiff(q, ws_result)

qtdiff(p, ws_result)

rtdiff(n, ws_result)
```

Arguments

`x, q`	Vector of quantiles
`ws_result`	Result from ws_tdiff_univariate()
`p`	Vector of probabilities
`n`	Number of observations

Value

For `dtdiff`: Numeric vector of density values. For `ptdiff`: Numeric vector of cumulative probabilities. For `qtdiff`: Numeric vector of quantiles. For `rtdiff`: Numeric vector of random samples from the approximated t-difference distribution.

Examples

```
result <- ws_tdiff_univariate(0, 1, 10, 0, 1.5, 15)
dtdiff(0, result)
ptdiff(0, result)
qtdiff(c(0.025, 0.975), result)
samples <- rtdiff(100, result)
```

validate_approximation *Validate Welch-Satterthwaite Approximation*

Description

Validates the approximation quality by comparing moments of the approximated distribution with the theoretical moments.

Usage

```
validate_approximation(ws_result, n_sim = 10000, seed = NULL)
```

Arguments

`ws_result`	Result from any ws_tdiff function
`n_sim`	Number of simulations for validation (default: 10000)
`seed`	Random seed for reproducibility

Value

A list containing validation metrics

Examples

```
result <- ws_tdiff_univariate(0, 1, 10, 0, 1.5, 15)
validation <- validate_approximation(result)
print(validation)
```

ws_tdiff_equal_params *Equal Parameters Special Case*

Description

Computes the Welch-Satterthwaite approximation for the special case where both distributions have identical parameters.

Usage

```
ws_tdiff_equal_params(mu, sigma, nu)
```

Arguments

`mu`	Common location parameter
`sigma`	Common scale parameter (must be > 0)
`nu`	Common degrees of freedom (must be > 4)

Details

When X1 ~ t(mu, sigma^2, nu) and X2 ~ t(mu, sigma^2, nu) are independent, the difference Z = X1 - X2 simplifies to:

- Location: mu_diff = 0
- Scale: sigma_star = sigma * sqrt(2*nu/(nu-2))
- Degrees of freedom: nu_star = 2*(nu - 4)

This special case provides validation for the general formulas and computational efficiency when parameters are known to be equal.

Value

An S3 object of class "ws_tdiff_univariate" with the simplified parameters

Examples

```
# Equal parameters case
result <- ws_tdiff_equal_params(mu = 0, sigma = 1, nu = 10)
print(result)
# nu_star should be 2*(10-4) = 12

# Verify against general formula
general <- ws_tdiff_univariate(0, 1, 10, 0, 1, 10)
all.equal(result$nu_star, general$nu_star)
```

`ws_tdiff_multivariate_general` *Welch-Satterthwaite Approximation for General Multivariate t-Differences*

Description

Approximates the distribution of differences between two independent multivariate t-distributed random vectors with arbitrary covariance structure. This implements Theorem 3 from Yamaguchi et al. (2025).

Usage

```
ws_tdiff_multivariate_general(
  mu1,
  Sigma1,
  nu1,
  mu2,
  Sigma2,
  nu2,
  max_iter = 10,
  tol = 0.001
)
```

Arguments

`mu1`	Location vector of first distribution (length p)
`Sigma1`	Scale matrix of first distribution (p x p, positive definite)
`nu1`	Degrees of freedom of first distribution (must be > 4)
`mu2`	Location vector of second distribution (length p)
`Sigma2`	Scale matrix of second distribution (p x p, positive definite)
`nu2`	Degrees of freedom of second distribution (must be > 4)
`max_iter`	Maximum iterations for convergence (default: 10)
`tol`	Convergence tolerance (default: 1e-6)

Details

This function handles the general case where components may be correlated within each multivariate t-distribution. The approximation uses a single scalar degrees of freedom parameter to capture the overall tail behavior.

The iterative algorithm (Section 4.3 of the paper):

1. Initialize with sum of covariance matrices
2. Compute effective degrees of freedom using trace formulas
3. Update scale matrix
4. Iterate until convergence

Note: For high dimensions with heterogeneous component behaviors, consider using `ws_tdiff_multivariate_independen` instead.

Value

An S3 object of class "ws_tdiff_multivariate_general" containing:

`mu_diff`	Location vector of difference
`Sigma_star`	Effective scale matrix
`nu_star`	Effective degrees of freedom (scalar)
`converged`	Logical indicating convergence
`iterations`	Number of iterations performed
`method`	Character string "multivariate_general"

Examples

```
Sigma1 <- matrix(c(1, 0.3, 0.3, 1), 2, 2)
Sigma2 <- matrix(c(1.5, 0.5, 0.5, 1.2), 2, 2)
result <- ws_tdiff_multivariate_general(
  mu1 = c(0, 1), Sigma1 = Sigma1, nu1 = 10,
  mu2 = c(0, 0), Sigma2 = Sigma2, nu2 = 15
)
print(result)
```

`ws_tdiff_multivariate_independent`
Welch-Satterthwaite Approximation for Multivariate t-Differences (Independent)

Description

Approximates the distribution of differences between two independent p-dimensional vectors with independent t-distributed components.

Usage

```
ws_tdiff_multivariate_independent(mu1, sigma1, nu1, mu2, sigma2, nu2)
```

Arguments

`mu1`	Location vector of first distribution (length p)
`sigma1`	Scale vector of first distribution (length p, all > 0)
`nu1`	Degrees of freedom vector of first distribution (length p, all > 4)
`mu2`	Location vector of second distribution (length p)
`sigma2`	Scale vector of second distribution (length p, all > 0)
`nu2`	Degrees of freedom vector of second distribution (length p, all > 4)

Details

This function applies the univariate Welch-Satterthwaite approximation component-wise when all components are mutually independent. Each component difference Zj = X1j - X2j is approximated independently using the univariate method.

This approach is optimal for:

- Marginal inference on specific components
- Cases where components have different tail behaviors
- Maintaining computational efficiency in high dimensions

Value

An S3 object of class "ws_tdiff_multivariate_independent" containing:

`mu_diff`	Location vector of difference
`sigma_star`	Vector of effective scale parameters
`nu_star`	Vector of effective degrees of freedom
`p`	Dimension of the vectors
`method`	Character string "multivariate_independent"

See Also

ws_tdiff_multivariate_general for correlated components

Examples

```
result <- ws_tdiff_multivariate_independent(
  mu1 = c(0, 1), sigma1 = c(1, 1.5), nu1 = c(10, 12),
  mu2 = c(0, 0), sigma2 = c(1.2, 1), nu2 = c(15, 20)
)
print(result)
```

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ws_tdiff_univariate *Welch-Satterthwaite Approximation for Univariate t-Differences*

Description

Approximates the distribution of the difference between two independent non-standardized t-distributed random variables using the Welch-Satterthwaite method.

Usage

```
ws_tdiff_univariate(mu1, sigma1, nu1, mu2, sigma2, nu2)
```

Arguments

mu1	Location parameter of first distribution
sigma1	Scale parameter of first distribution (must be > 0)
nu1	Degrees of freedom of first distribution (must be > 4)
mu2	Location parameter of second distribution
sigma2	Scale parameter of second distribution (must be > 0)
nu2	Degrees of freedom of second distribution (must be > 4)

Details

For two independent non-standardized t-distributed random variables:

- X1 ~ t(mu1, sigma1^2, nu1)
- X2 ~ t(mu2, sigma2^2, nu2)

The difference Z = X1 - X2 is approximated as: Z ~ t(mu1 - mu2, sigma_star^2, nu_star)

where the effective parameters are computed through moment matching:

- sigma_star matches the variance of Z
- nu_star is derived from fourth moment matching

The method requires nu1 > 4 and nu2 > 4 for the existence of fourth moments. The approximation quality improves as degrees of freedom increase and approaches exactness as nu -> infinity (normal limit).

Value

An S3 object of class "ws_tdiff_univariate" containing:

`mu_diff`	Location parameter of difference (mu1 - mu2)
`sigma_star`	Effective scale parameter (Equation 1 from paper)
`nu_star`	Effective degrees of freedom (Equation 2 from paper)
`input_params`	List of input parameters for reference
`method`	Character string "univariate"

References

Yamaguchi, Y., Homma, G., Maruo, K., & Takeda, K. Welch-Satterthwaite Approximation for Difference of Non-Standardized t-Distributed Variables. (unpublished).

See Also

`ws_tdiff_equal_params` for the special case of equal parameters `dtdiff`, `ptdiff`, `qtdiff`, `rtdiff` for distribution functions

Examples

```
# Example 1: Different scale parameters
result <- ws_tdiff_univariate(
  mu1 = 0, sigma1 = 1, nu1 = 10,
  mu2 = 0, sigma2 = 1.5, nu2 = 15
)
print(result)

# Example 2: Equal parameters (special case)
result_equal <- ws_tdiff_univariate(
  mu1 = 5, sigma1 = 2, nu1 = 20,
  mu2 = 3, sigma2 = 2, nu2 = 20
)
# Should match ws_tdiff_equal_params(5-3, 2, 20)
```

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