Complete formulas used by `coverage`

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Function `coverage` of **actuar** defines a new function to compute the probability density function (pdf) of cumulative distribution function (cdf) of any probability law under the following insurance coverage modifications: ordinary or franchise deductible, limit, coinsurance, inflation.

In addition, the function can return the distribution of either the payment per loss or the payment per payment random variable. This terminology refers to whether or not the insurer knows that a loss occurred. For the exact definitions of the terms as used by `coverage`, see Chapter 5 of Klugman et al. (2004).

In the presence of a deductible, four random variables can be defined:

1. Y^P, the payment per payment with an ordinary deductible;
2. Y^L, the payment per loss with an ordinary deductible;
3. $\tilde{Y}^P$, the payment per payment with a franchise deductible;
4. $\tilde{Y}^L$, the payment per loss with a franchise deductible.

The most common case in insurance applications is the distribution of the amount paid per payment with an ordinary deductible, Y^P. Hence, it is the default in `coverage`.

When there is no deductible, all four random variables are equivalent.

This document presents the definitions of the above four random variables and their corresponding cdf and pdf for a deductible d, a limit u, a coinsurance level α and an inflation rate r. An illustrative plot of each cdf and pdf is also included. In these plots, a dot indicates a probability mass at the given point.

In definitions below, X is the nonnegative random variable of the losses with cdf $F_X(\cdot)$ and pdf $f_X(\cdot)$.

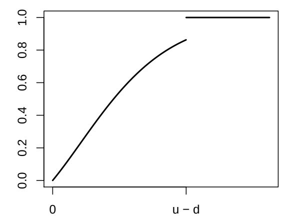
References

S. A. Klugman, H. H. Panjer, and G. Willmot. *Loss Models: From Data to Decisions*. Wiley, New York, 2 edition, 2004. ISBN 0-4712157-7-5.

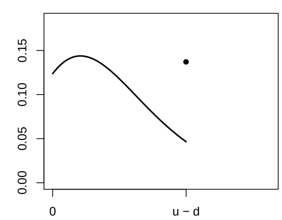
1 Payment per payment, ordinary deductible

$$Y^P = \begin{cases} \alpha((1+r)X - d), & \dfrac{d}{1+r} \leq X < \dfrac{u}{1+r} \\ \alpha(u-d), & X \geq \dfrac{u}{1+r} \end{cases}$$

$$F_{Y^P}(y) = \begin{cases} 0, & y = 0 \\ \dfrac{F_X\left(\frac{y+\alpha d}{\alpha(1+r)}\right) - F_X\left(\frac{d}{1+r}\right)}{1 - F_X\left(\frac{d}{1+r}\right)}, & 0 < y < \alpha(u-d) \\ 1, & y \geq \alpha(u-d) \end{cases}$$



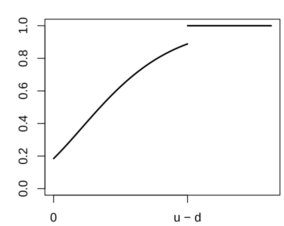
$$f_{Y^P}(y) = \begin{cases} 0, & y = 0 \\ \left(\dfrac{1}{\alpha(1+r)}\right) \dfrac{f_X\left(\frac{y+\alpha d}{\alpha(1+r)}\right)}{1 - F_X\left(\frac{d}{1+r}\right)}, & 0 < y < \alpha(u-d) \\ \dfrac{1 - F_X\left(\frac{u}{1+r}\right)}{1 - F_X\left(\frac{d}{1+r}\right)}, & y = \alpha(u-d) \end{cases}$$



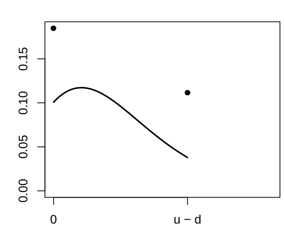
2 Payment per loss, ordinary deductible

$$Y^L = \begin{cases} 0, & X < \dfrac{d}{1+r} \\ \alpha((1+r)X - d), & \dfrac{d}{1+r} \leq X < \dfrac{u}{1+r} \\ \alpha(u-d), & X \geq \dfrac{u}{1+r} \end{cases}$$

$$F_{Y^L}(y) = \begin{cases} F_X\left(\dfrac{d}{1+r}\right), & y = 0 \\ F_X\left(\dfrac{y+\alpha d}{\alpha(1+r)}\right), & 0 < y < \alpha(u-d) \\ 1, & y \geq \alpha(u-d) \end{cases}$$



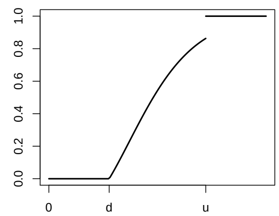
$$f_{Y^L}(y) = \begin{cases} F_X\left(\dfrac{d}{1+r}\right), & y = 0 \\ \dfrac{1}{\alpha(1+r)} f_X\left(\dfrac{y+\alpha d}{\alpha(1+r)}\right), & 0 < y < \alpha(u-d) \\ 1 - F_X\left(\dfrac{u}{1+r}\right), & y = \alpha(u-d) \end{cases}$$



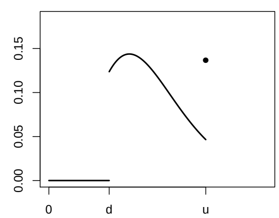
3 Payment per payment, franchise deductible

$$\tilde{Y}^P = \begin{cases} \alpha(1+r)X, & \dfrac{d}{1+r} \le X < \dfrac{u}{1+r} \\ \alpha u, & X \ge \dfrac{u}{1+r} \end{cases}$$

$$F_{\tilde{Y}^P}(y) = \begin{cases} 0, & 0 \le y \le \alpha d \\ \dfrac{F_X\left(\frac{y}{\alpha(1+r)}\right) - F_X\left(\frac{d}{1+r}\right)}{1 - F_X\left(\frac{d}{1+r}\right)}, & \alpha d < y < \alpha u \\ 1, & y \ge \alpha u \end{cases}$$



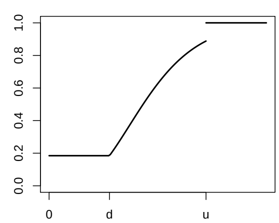
$$f_{\tilde{Y}^P}(y) = \begin{cases} 0, & 0 \le y \le \alpha d \\ \left(\dfrac{1}{\alpha(1+r)}\right) \dfrac{f_X\left(\frac{y}{\alpha(1+r)}\right)}{1 - F_X\left(\frac{d}{1+r}\right)}, & \alpha d < y < \alpha u \\ \dfrac{1 - F_X\left(\frac{u}{1+r}\right)}{1 - F_X\left(\frac{d}{1+r}\right)}, & y = \alpha u \end{cases}$$



4 Payment per loss, franchise deductible

$$\tilde{Y}^L = \begin{cases} 0, & X < \dfrac{d}{1+r} \\ \alpha(1+r)X, & \dfrac{d}{1+r} \le X < \dfrac{u}{1+r} \\ \alpha u, & X \ge \dfrac{u}{1+r} \end{cases}$$

$$F_{\tilde{Y}^L}(y) = \begin{cases} F_X\left(\dfrac{d}{1+r}\right), & 0 \le y \le \alpha d \\ F_X\left(\dfrac{y}{\alpha(1+r)}\right), & \alpha d < y < \alpha u \\ 1, & y \ge \alpha u \end{cases}$$



$$f_{\tilde{Y}^L}(y) = \begin{cases} F_X\left(\dfrac{d}{1+r}\right), & y = 0 \\ \dfrac{1}{\alpha(1+r)} f_X\left(\dfrac{y}{\alpha(1+r)}\right), & \alpha d < y < \alpha u \\ 1 - F_X\left(\dfrac{u}{1+r}\right), & y = \alpha u \end{cases}$$